In order to discover how far the experiment supports either of these positions, Mr. Donovan adduced counter-experiments, in which combinations of zinc and copper were acted on by dilute acid of different strengths until dissolved. The solution took place in different periods of time, and, consequently, the electricity evolved during any given period was unequal in quantity, in some cases very much so; yet in all of them the effect on the galvanometer was the same.

These experiments appear incompatible with Faraday's law of equal quantities of electricity producing equal deflections, irrespectively of other circumstances. Support is, consequently, withdrawn by them from his estimate of the enormous quantity of electricity naturally associated with matter.

The following note by Professor Mac Cullagh was read.

Let a surface A of the second order be represented by the equation

$$\frac{x^2}{P_0} + \frac{y^2}{Q_0} + \frac{z^2}{R_0} = 1,$$

its primary axis being that of x. Through a given point S, whose coordinates are x', y', z', conceive three surfaces confocal with A to be described, and let P, P', P'' be the squares of their primary semiaxes. Then, if normals drawn to these surfaces respectively at the point S be the axes of a new system of coordinates ξ, η, ζ, and if we put

$$P - P_0 = k, \qquad P' - P_0 = k', \qquad P'' - P_0 = k'',$$

$$\frac{x'^2}{P_0} + \frac{y'^2}{Q_0} + \frac{z'^2}{R_0} = f,$$

the equation of the surface A, referred to the new coordinates, will be

$$\frac{\xi^2}{k} + \frac{\eta^2}{k'} + \frac{\zeta^2}{k''} = (f - 1)\left(\frac{\xi_0\xi}{k} + \frac{\eta_0\eta}{k'} + \frac{\zeta_0\zeta}{k''} - 1\right)^2, \qquad (a)$$

where ξ_0, η_0, ζ_0 are the coordinates of its centre.

From the form of this equation it is evident, that if the surface be intersected by the plane whose equation is

$$\frac{\xi_0\xi}{k} + \frac{\eta_0\eta}{k'} + \frac{\zeta_0\zeta}{k''} = 1, \qquad (b)$$

it will be touched along the curve of intersection by the cone whose equation is

$$\frac{\xi^2}{k} + \frac{\eta^2}{k'} + \frac{\zeta^2}{k''} = 0. \qquad (c)$$

This mode of deducing, in its simplest form, the equation of a cone circumscribing a surface of the second order, is much easier than the direct investigation by which the equation (c) was originally obtained.

Let a right line passing through S intersect the plane expressed by the equation (b), in a point whose distance from S is equal to ϖ, while it intersects the surface A in two points, P and P$'$, the distance of either of which from S is denoted by ρ. Let the surface B, represented by the equation

$$\frac{\xi^2}{k} + \frac{\eta^2}{k'} + \frac{\zeta^2}{k''} = f - 1, \qquad (d)$$

be intersected by the same right line in a point whose distance from S is equal to r, the distance r being, of course, a semidiameter of this surface. Then it is obvious that the equation (a) may be written

$$\frac{1}{r^2} = \left(\frac{1}{\varpi} - \frac{1}{\rho}\right)^2;$$

so that, if ρ and ρ' represent the distances SP and SP$'$ respectively, we have

$$\frac{1}{\rho} = \frac{1}{\varpi} \times \frac{1}{r}, \qquad \frac{1}{\rho'} = \frac{1}{\varpi} - \frac{1}{r}; \qquad (e)$$

and therefore

$$\frac{1}{\rho} - \frac{1}{\rho'} = \frac{2}{r}. \qquad (f)$$

This result is useful in questions relating to attraction. For if A be an ellipsoid, every point of which attracts an external point S with a force varying inversely as the *fourth* power of the distance, and if the point S be the vertex of a pyramid, one of whose sides is the right line SPP′, and whose transverse section, at the distance unity from its vertex, is the indefinitely small area ω, the portion PP′ of the pyramid will attract the point S, in the direction of its length, with a force expressed by the quantity

$$\left(\frac{1}{\rho}-\frac{1}{\rho'}\right)\omega, \quad \text{or } \frac{2\omega}{r};$$

and, putting θ for the angle which the right line SP makes with the axis of ξ, the attraction in the direction of ξ will be

$$\frac{2\omega\cos\theta}{r}. \qquad (g)$$

Now, supposing the axis of ξ to be normal to the confocal ellipsoid described through S, it will be the primary axis of the surface B, which will be a hyperboloid of two sheets; and, the surface being symmetrical round this axis, it is easy to see, from the expression for the elementary attraction, that the whole attraction of the ellipsoid will be in the direction of ξ. Therefore, when the force is inversely as the fourth power of the distance, the attraction of an ellipsoid on an external point is normal to the confocal ellipsoid passing through that point.

Hence we infer, that if υ be the sum of the quotients found by dividing every element of the volume of an ellipsoid by the cube of its distance from an external point, the value of υ will remain the same, wherever that point is taken on the surface of an ellipsoid confocal with the given one.

The question of the attraction of an ellipsoid, when the law of force is that of the inverse square of the distance, has been treated by Poisson, in an elegant but very elaborate memoir, presented to the Academy of Sciences in 1833 (*Mé-*

moires de l'Institut, tom. xiii.) In the preceding year I had obtained the theorems just mentioned, by considering the law of the inverse fourth power; and, as well as I remember, they were deduced exactly as above, by setting out from the equation (a). But I did not then succeed in applying the same method to the case where the law of force is that of nature, probably from not perceiving that, in this case, the ellipsoid ought to be divided (as Poisson has divided it) into concentric and similar shells. This application requires the following theorem, which is easily proved:

Supposing A′ to be another ellipsoid, concentric, similar, and similarly placed with A, let the right line SPP′ intersect it in the points p and p', respectively adjacent to P and P′; then, if the direction of that right line be conceived to vary, the rectangle under Pp and P′p (or under Pp' and P′p') will be to the rectangle under SP and SP′ in a constant ratio.

Denoting the constant ratio by m, and combining this theorem with the formula (f), we have

$$\frac{\mathrm{P}p \times \mathrm{P}'p}{\mathrm{PP}'} = \frac{mr}{2}. \qquad (h)$$

Now let the two surfaces A and A′ be supposed to approach indefinitely near each other, so as to form a very thin shell, then ultimately P′p will be equal to PP′, and we shall have

$$\mathrm{P}p = \mathrm{P}'p' = \frac{mr}{2},$$

where m is indefinitely small. Therefore, if the point S, external to the shell, be the vertex of a pyramid whose side is the right line SP, and whose section, at the unit of distance from the vertex, is ω, the attraction of the two portions Pp and P′p' of this pyramid, which form part of the shell, will be equal to $mr\omega$. Hence it appears, as before, on account of the symmetry of the surface B round the axis of ξ, that the whole attraction of the shell on the point S is in the direction of that axis, and consequently (as was found by Poisson) in the direction

of the internal axis of the cone whose vertex is S, and which circumscribes the shell.

To find the whole attraction of the shell, the expression

$$mr\omega \cos \theta \qquad (i)$$

must be integrated. Let ϕ be the angle which a plane, passing through SP and the axis of ξ, makes with the plane $\xi\eta$; then

$$\omega = \sin \theta d\theta d\phi,$$

$$\frac{1}{r} = \sqrt{\left(\frac{\cos^2\theta}{k} + \frac{\sin^2\theta \cos^2\phi}{k'} + \frac{\sin^2\theta \sin^2\phi}{k''}\right)} \frac{1}{\sqrt{f-1}}.$$

When these values are substituted in (i), that expression may be readily integrated, first with respect to θ, and then with respect to ϕ.

It is evident that, by the same substitutions, the expression (g) may be twice integrated.

An investigation similar to the preceding has been given by M. Chasles, for the case in which the force varies inversely as the square of the distance (*Mémoires des Savants Etrangers*, tom. ix.) He uses a theorem equivalent to the formula (f), but deduces it in a different way.

From what has been proved it follows that, if v be the sum of the quotients found by dividing every element of the shell by its distance from an external point S, the value of v will be the same wherever that point is taken on the surface Σ of an ellipsoid confocal with the surface A of the shell.

Let Σ' be another ellipsoid confocal with A, and indefinitely near the surface Σ. The normal interval between the two surfaces Σ and Σ', at any point S on the former, will be inversely as the perpendicular dropped from the common centre of the ellipsoids on the plane which touches Σ at S. Hence, supposing the point S to move over the surface Σ, that perpendicular will vary as the attraction exerted by the shell on the point S, when the force is inversely as the square of the

distance, or as the attraction exerted by the whole ellipsoid A on the point S, when the force is inversely as the fourth power of the distance.

When the point S is on the focal hyperbola, the integrations, by which the actual attraction is found in either case, are simplified, for the surface B is then one of revolution round the axis of ξ, and its semidiameter r is independent of the angle ϕ.

From the expression for the attraction of a shell we can find, by another integration, the attraction of the entire ellipsoid, when the law of force is that of nature. And thus the well-known problem of the integral calculus, in which it is proposed to determine directly the attraction of an ellipsoid on an external point, without employing the theorem of Ivory to evade the difficulty, is solved in what appears to be the simplest manner.

The preceding note having been read, Mr. Graves observed that the mention therein made, of the equation which represents so simply a cone circumscribing a given surface of the second order, reminded him of a circumstance which he thought it right to state; as that remarkable equation had been in circulation among geometers long before it appeared in print, and thus its origin, though generally known, was sometimes mistaken. Mr. Graves stated that he still retains a large part of the memoranda, in which he set down, from day to day, the substance of Professor Mac Cullagh's lectures, delivered in Hilary Term, 1836, and that the part preserved contains the equation in question (the equation (c) of the preceding note). In the memoranda it is deduced directly; that is, the equation of the cone is first given in the usual form, and is then reduced to the form (c) by a transformation of coordinates.